

Basic Mathematics

by physics in short

- **Quadratic equation (द्विघात समीकरण) $ax^2+bx+c=0$ के solutions (हल) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$**

Note : यदि $b^2 - 4ac < 0$ तो roots imaginary (काल्पनिक) होंगे एवं यदि $b^2=4ac$ तो roots equal होंगे।

Sum of roots = $x_1 + x_2 = -b/a$, Product of roots = $x_1 x_2 = c/a$

Exercise : Solve : $10x^2 - 27x + 5 = 0$ **Ans.** $x = 5/2, x = 1/5$

- **Binomial Theorem (द्विपद प्रमेय)**

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots$$

if $x \ll 1$ [$x, 1$ से बहुत छोटा है] तो **$(1 + x)^n \approx 1 + nx$**

Remember Always

$$\sqrt{2}=1.41, \sqrt{3}=1.73, \sqrt{5}=2.23$$

$$2^{1/3} = 1.26, 3^{1/3} = 1.44,$$

$$4^{1/3} = 1.59, 5^{1/3} = 1.71$$

- **Trigonometry (त्रिकोणमिति)**

$$\sin^2\theta + \cos^2\theta = 1 \quad \sin \theta = \frac{a}{\sqrt{a^2 + b^2}} = \frac{1}{\operatorname{cosec} \theta}$$

$$1 + \tan^2\theta = \sec^2\theta \quad \cos \theta = \frac{b}{\sqrt{a^2 + b^2}} = \frac{1}{\sec \theta}$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta \quad \tan \theta = \frac{a}{b} = \frac{\sin \theta}{\cos \theta} = \frac{1}{\cot \theta}$$

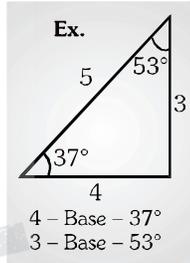
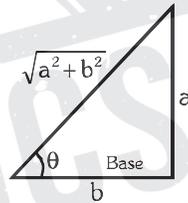
$$\sin 2\theta = 2 \sin \theta \cos \theta; \cos 2\theta = \cos^2\theta - \sin^2\theta = 1 - 2 \sin^2\theta = 2 \cos^2\theta - 1$$

$$\pi \text{ radian} = 180^\circ \Rightarrow 1 \text{ rad} \approx 57^\circ \& \quad 1^\circ = 60' \text{ (मिनट)}; \quad 90^\circ = \pi/2 \text{ rad.}$$

$$\sin(-\theta) = -\sin \theta; \quad \cos(-\theta) = +\cos \theta \quad \tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin \theta}{\cos \theta} = -\tan \theta$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \quad \cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B \quad \cos(A - B) = \cos A \cos B + \sin A \sin B$$



Vectors (सदिश)

- **Electric current (विद्युत धारा) is not a vector**

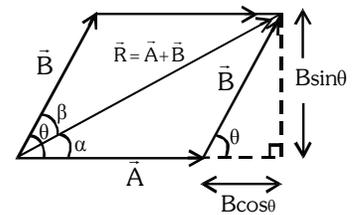
[Although it has both magnitude and direction but it does not follow triangle rule of addition]

- **Addition of vectors (सदिशों का योग)**

$$|\vec{R}| = R = |\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2\vec{A} \cdot \vec{B}} = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$\text{परिणामी (Resultant) का } \vec{A} \text{ के साथ कोण (Angle) } \tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\text{परिणामी (Resultant) का } \vec{B} \text{ के साथ कोण (Angle) } \tan \beta = \frac{A \sin \theta}{B + A \cos \theta}$$



यदि $|\vec{A}| = |\vec{B}|$ तो $|\vec{R}| = 2A \cos \theta/2$ इस स्थिति में $\tan \alpha = \tan \beta = \tan \theta/2$ [or $\alpha = \beta = \theta/2$]

■ **Subtraction of vectors (सदिशों का घटाना)**

$$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \text{ यहाँ } |\vec{R}| = \sqrt{A^2 + B^2 + 2AB \cos(\pi - \theta)} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$\text{यदि } |\vec{A}| = |\vec{B}| \text{ तो } |\vec{R}| = 2A \sin \frac{\theta}{2} \text{ इस स्थिति में } \tan \alpha = \tan \beta = \cot \frac{\theta}{2} \left[\text{or } \alpha = \beta = \frac{\pi}{2} - \frac{\theta}{2} \right]$$

- किसी सदिश (vector) को positive scalar (धनात्मक अदिश) से गुणा करने पर दिशा वही रहेगी परन्तु magnitude (परिमाण) बदल जायेगा। जैसे $3\vec{A}$, इसकी दिशा \vec{A} की दिशा में ही होगी जबकि magnitude, \vec{A} के magnitude का तीन गुना है। Negative scalar से multiply करने पर direction opposite हो जाती है। जैसे $(-4)\vec{A} = -4\vec{A}$ इसकी दिशा \vec{A} की दिशा के विपरीत जबकि परिमाण \vec{A} के परिमाण का 4 गुना होगा।

■ **Dot (Scalar) product [बिन्दु (अदिश) गुणनफल]**

$$\vec{A} \cdot \vec{B} = AB \cos \theta \equiv \text{एक अदिश (a scalar)}$$

$$\text{यदि } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \text{ एवं } \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \text{ तो } \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

■ **Cross (Vector) product [वज्र (सदिश) गुणनफल]**

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} \equiv \text{एक सदिश (a vector)}$$

$\hat{n} \rightarrow$ unit vector perpendicular to both \vec{A} and \vec{B} (\vec{A} व \vec{B} दोनों के लम्बवत एकांक सदिश)

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}[A_y B_z - B_y A_z] - \hat{j}[A_x B_z - B_x A_z] + \hat{k}[A_x B_y - B_x A_y]$$

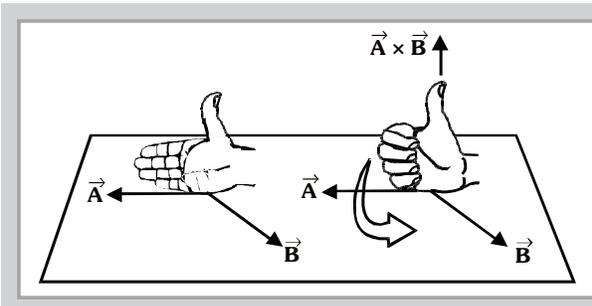
$$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - B_y A_z) + \hat{j}(A_z B_x - B_z A_x) + \hat{k}(A_x B_y - B_x A_y) \text{ इसको याद रखें।}$$

$$\vec{A} \times \vec{B} = \hat{x}(A_y B_z - B_y A_z) + \hat{y}(A_z B_x - B_z A_x) + \hat{z}(A_x B_y - B_x A_y) \text{ इसको ऐसे भी याद रखा जा सकता है।}$$

Ex. If $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{B} = -3\hat{i} + 6\hat{k}$ then calculate $\vec{A} \times \vec{B}$

Sol. $\vec{A} \times \vec{B} = \hat{i}[(-3)(6) - (0)(4)] + \hat{j}[(4)(-3) - (6)(2)] + \hat{k}[(2)(0) - (-3)(-3)] = -18\hat{i} - 24\hat{j} - 9\hat{k}$

■ **Right hand thumb rule**



पहले vector \vec{A} (सदिश) की तरफ fingers (अंगुलियों) को रखकर दूसरे vector \vec{B} की तरफ छोटे angle की दिशा में घुमाने पर thumb (अंगूठे) की direction $\vec{A} \times \vec{B}$ की direction होगी।

Note : $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$
(दोनों की direction opposite होगी)

Important Conversions

Plane angle (समतल कोण)	1 degree = 60 minute = 3600 second = $2\pi/360$ radian = $1/360$ revolution 1 radian = 57.3° , 1 revolution (1 चक्कर) = $360^\circ \equiv 2\pi$ radian
Length (लम्बाई)	100 centimeter = 1 metre = 1000mm ; 1 inch = 2.54 cm 1 foot=30.48 cm, 1 mile (1 मील)=1609 m, $1\text{\AA}=10^{-10}$ m, 1 fermi= 10^{-15} m 1 nautical mile (1 समुद्री मील)=1852 m, (Note : 1 knot=1.852 km/hr), 1 yard (1 गज)=3ft
1 parsec > 1 light year > 1 AU	1 light year (1 प्रकाश वर्ष = प्रकाश द्वारा एक वर्ष में चली दूरी) = 9.46×10^{12} km 1 parsec (1 पारसेक) = 3.26 light year 1AU (astronomical unit) = 1.496×10^{11} m $\approx 1.5 \times 10^{11}$ m =सूर्य एवं पृथ्वी के बीच की दूरी 1 bohr radius = 0.529\AA , 1 mil = 10^{-3} inch, 1 nm = 10^{-9} m, 1 μ m = 10^{-6} m, 1 mm = 10^{-3} m, 1 pm = 10^{-12} m
Area (क्षेत्रफल)	1 cm ² = 10^{-4} m ² , 1 barn = 10^{-28} m ² , 1 hectare= 10^4 m ² = 2.47 acres (एकड़)
Speed (चाल)	1 km / hr = $5/18$ m/s, 1 mile/hr = 0.45 m/s
Volume (आयतन)	1 m ³ = 10^6 cm ³ = 10^6 cc = 1000 litre = 35.31 ft ³ (घनफीट)
Mass (द्रव्यमान)	1 atomic mass unit (a.m.u.) = 1.66×10^{-27} kg = 1u 1 pound = 453.6 g, 1 ounce = 28.35 g 1 slug = 14.6 kg, 1 ton = 907.2 kg, 1 metric ton = 1000 kg *Chandra shekhar limit = $1.4 M_s$ where M_s = Mass of sun
Force (बल)	10^5 dyne = 1 newton = 0.225 lb (1 lb = 4.45 N) 1 gram-force = 980 dyne, 1 kilogram force (1kgf) = 9.8 N
Acceleration (त्वरण)	$g = 9.8 \text{ m/sec}^2$ (MKS unit) $\equiv 980 \text{ cm/sec}^2$ (CGS unit)= 32 feet/sec^2 (FPS unit)
Pressure (दाब)	1 atm (1 वायुमण्डलीय दाब)= 1.013×10^6 dyne/cm ² = 1.013×10^5 N/m ² = 1.013×10^5 pascal=76 cm Hg=760mm Hg { \therefore 1pascal = 1newton/m ² } 1 bar = 10^5 N/m ² = 10^6 dyne/cm ² ; 1 torr = 1 mm Hg
Density (घनत्व)	1 kg/m ³ = 10^{-3} g/cm ³
Time (समय)	1 day = 86,400 second, 1 year = $365 \frac{1}{4}$ days = 3.16×10^7 second
Energy & Power (ऊर्जा तथा शक्ति)	1 Btu (British thermal unit) = 1055 joule; 1 joule = 10^7 erg , 1 cal = 4.18 J 1 eV = 1.6×10^{-19} J, 1 horsepower (hp) = 746 watt
Electric charge (विद्युत आवेश)	1 coulomb $\Leftrightarrow 2.998 \times 10^9$ statcoulombs (or esu) of charge $\approx 3 \times 10^9$ stC $\Leftrightarrow 0.1$ abcoulomb (or emu of charge) (Note : esu – electrostatic unit, emu – electromagnetic unit)
Electric current (विद्युत धारा)	1 ampere $\Leftrightarrow 2.998 \times 10^9$ statampere (or esu of current) $\Leftrightarrow 0.1$ abampere (or emu of current)
Electric potential (विद्युत विभव)	1 volt $\Leftrightarrow 3.336 \times 10^{-3}$ statvolt $\Leftrightarrow 1 \times 10^8$ abvolts 1 statvolt = 300 volts
Electric field (विद्युत क्षेत्र)	1 volt per meter $\Leftrightarrow 3.336 \times 10^{-5}$ statvolt/cm $\Leftrightarrow 1 \times 10^6$ abvolt/cm
Magnetic field (चुम्बकीय क्षेत्र)	1 T = 10^4 G, 1 Wb = 10^8 maxwell, 1T=1 Wb/m ² , 1G = 1 maxwell/cm ²

Some Important Dimensional Formulae

- **Relative Density or Specific Gravity** (आपेक्षिक घनत्व या विशिष्ट गुरुत्व)

$$= \frac{\text{Density of substance (पदार्थ का घनत्व)}}{\text{Density of water (पानी का घनत्व)}} = \frac{ML^{-3}}{ML^{-3}} = M^0L^0T^0$$
No dimension (विमाहीन)
- **Mechanical Equivalent of heat** (ऊष्मा का यांत्रिक तुल्यांक)

$$J = \frac{\text{Mechanical Work done (यांत्रिक कार्य)}}{\text{Heat produced (उत्पन्न ऊष्मा)}} = \frac{ML^2T^{-2}}{ML^2T^{-2}} = M^0L^0T^0$$
No dimension
- **Compressibility** (सम्पीड्यता) $= \frac{1}{\text{Bulk modulus}} = \frac{1}{K} = \frac{1}{ML^{-1}T^{-2}} = M^{-1}LT^2$ **[M⁻¹LT²]**
- **Avogadro Number** (आवोगाद्रो संख्या) $(N_A) = N_A/\text{mole}$ **[mol⁻¹] or [μ⁻¹]**
- **Gas Constant** (गैस नियतांक) $R = \frac{PV}{\mu T} = \frac{ML^{-1}T^{-2} \times L^3}{\theta \times \text{mol}} = ML^2T^{-2}\theta^{-1} \text{ mol}^{-1}$ **[ML²T⁻²θ⁻¹ mol⁻¹]**
- **Boltzmann's Constant** $k = \frac{R(\text{Gas Constant})}{N(\text{Avogadro number})} = \frac{ML^2T^{-2}\theta^{-1} \text{ mol}^{-1}}{\text{mol}^{-1}}$ **[ML²T⁻²θ⁻¹]**
- **Vander Waal's Constants** $\therefore \left(P + \frac{\mu^2 a}{V^2}\right)(V - \mu b) = \mu RT$ $\therefore \frac{\mu^2 a}{V^2} = \text{Pressure (dimensionally)}$
 $\Rightarrow a = \frac{PV^2}{\mu^2} = \frac{ML^{-1}T^{-2}}{\text{mol}^2} \times (L^3)^2 = ML^5T^{-2} \text{ mol}^{-2}$ **[ML⁵T⁻²mol⁻²]** and $b = \frac{\text{volume}}{\mu} = \frac{L^3}{\mu}$ **[L³μ⁻¹]**
- **Entropy** (एन्ट्रॉपी) $= \frac{\text{Amount of Heat (ऊष्मा की मात्रा)}}{\text{Temperature (ताप)}} = \frac{ML^2T^{-2}}{\theta} = ML^2T^{-2}\theta^{-1}$ **[ML²T⁻²θ⁻¹]**
- **L/R or RC** → Time constant of L-R/R-C circuit (समय नियतांक) = dimension of time = T **[T]**
- **Power of Lens** (लेंस की शक्ति) $P = \frac{1}{f} = \frac{1}{f \text{ metre}} = \frac{100}{f \text{ cm}} = \frac{1}{L} = L^{-1}$ **[L⁻¹]**
 (Note : for lens $P = 1/f$ and for mirrors $P = -1/f$)
- **Magnification** (आवर्धन) {for lens $m = \frac{v}{u}$ & for mirrors $m = -\frac{v}{u}$ } **(No dimension)**
- **Enthalpy** (एन्थैल्पी) $\therefore H = U + PV = \text{Total heat content} = ML^2T^{-2}$ **[ML²T⁻²]**
- **Coefficient of Viscosity** (श्यानता गुणांक) : $\eta = F/6\pi r v$ **[ML⁻¹T⁻¹]**

Conversion of Units from one system to another

Use $n_2 = n_1 \left(\frac{M_1}{M_2}\right)^a \left(\frac{L_1}{L_2}\right)^b \left(\frac{T_1}{T_2}\right)^c$ where $M^a L^b T^c$ is the dimensional formula of physical quantity

[∴ nu = constant संख्यात्मक मान × मात्रक = मान = नियत]

Ex. $F = MLT^{-2}$ हम यहाँ force की SI (MKS) तथा CGS system में value को compare करेंगे।

$$n_2(\text{CGS}) = n_1(\text{MKS}) \left(\frac{M_1}{M_2}\right)^1 \left(\frac{L_1}{L_2}\right)^1 \left(\frac{T_1}{T_2}\right)^{-2} = 1 \left(\frac{1\text{kg}}{1\text{g}}\right) \left(\frac{1\text{metre}}{1\text{cm}}\right) \left(\frac{1\text{sec}}{1\text{sec}}\right)^{-2} = \left(\frac{1000\text{g}}{1\text{g}}\right) \left(\frac{100\text{cm}}{1\text{cm}}\right) = 10^5$$

FBD Technique (NLM)

B

See:

- Question of **Rest** — Equilibrium
 - $\Sigma F=0$
 - $\Sigma \tau=0$
- Question of **Motion**

R

Check :

forces involved in the given question

- Weight
- Tension
- Contact force
 - Normal force
 - Frictional force (f)
 - f_s
 - $(f)_{normal}$
 - $(f)_{limiting}$
 - f_k
 - Spring force
 - Thrust

A

Decide :

Draw **FBD** (depends on observers position)

- w.r.t. Ground \Rightarrow No concept of pseudo force (Inertial frame)
- w.r.t. accelerated frame \Rightarrow Apply concept of pseudo force (non-inertial frame)

I

N

Directions:

- Horizontal & Vertical
- Parallel to inclined plane & normal to I.P.
- Radial (normal) & tangential.
- NEWS (NLE, WLS)
- $(x \perp y), (y \perp z) (z \perp x)$

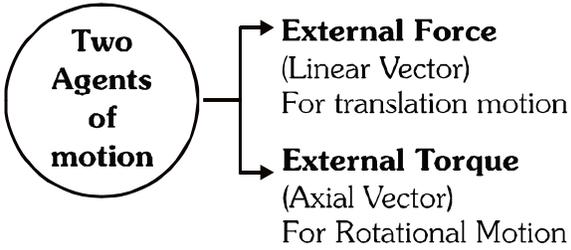
P

Start :

N

In drawing FBD firstly show that force which appear as Action-Reaction pair between the bodies. जिस force से body, system से अलग होकर free body बनी है, उसे सबसे पहले दिखाओ।

Dynamics of Motion (Cause of motion)



NEWTON MECHANICS

Basically five forces in Newton Mechanics : Field \Rightarrow Force \Rightarrow Acceleration \Rightarrow Kinematics

❶ **Weight (भार)** : \downarrow (नीचे की ओर) Due to gravity of Earth (गुरुत्वीय बल)

❷ **Tension (तनाव)** : Always away (हमेशा दूर) from body
(Due to stretching)

- Living (जीवित)
- Nonliving (अजीवित)

☺ **Tension** हमेशा हमसे कोसों दूर होना चाहिए।

❸ **Contact Force** (सम्पर्क बल) (due to pressing) (दबाव के कारण)

- Normal Contact Force (Perpendicular to contact surface towards the body)
- Tangential contact force : Frictional force (Parallel to contact surfaces)

} Electromagnetic nature

❹ **Spring Force (स्प्रिंग बल)**

$U = \frac{1}{2} K(\Delta \ell)^2$

❺ **Thrust Force (Bouyant Force) (उत्प्लावन बल)** Always in upward direction

[Magnitude = $V\rho g$ where V =Volume of body, ρ = Density of medium].



Relative Motion

$$\vec{x}_{Ab} = \vec{x}_{Rel} + \vec{x}_{ref} \text{ where } \vec{x} \rightarrow \vec{r}, \vec{v}, \vec{a}, \vec{\omega}, \vec{p}$$

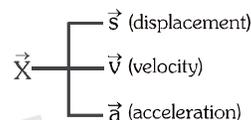
<p>पहचान करना (कौन किसके relative)</p>	<p>बोलना (तोल-मोल के बोलना)</p> \vec{x}_{AB} या \vec{x}_{BA}	<p>लिखना (धैर्य से लिखना)</p>	<p>ज्ञात करना (धैर्य से करना)</p>	<p>best approach $\hat{i} \hat{j} \hat{k}$ vector addition rule</p>	<p>मस्त रहना (answer का मिलना)</p>
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Relative quantity = Actual quantity - Reference quantity (Actual = Ref + Relative)

$\vec{x}_{relative}$ & \vec{x}_{actual} → जिसको describe किया जा रहा है। $\vec{x}_{reference}$ → जिसके respect में describe किया जा रहा है।

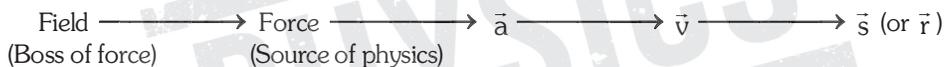
$$\vec{x}_{relative} = \vec{x}_{actual} - \vec{x}_{reference}$$

$$\vec{x}_{actual} = \vec{x}_{reference} + \vec{x}_{relative}$$



Ex. $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B \Rightarrow \vec{v}_A = \vec{v}_B + \vec{v}_{AB}$

Calculus Approach



x-direction	F _x	a _x	v _x	x
y-direction	F _y	a _y	v _y	y
z-direction	F _z	a _z	v _z	z

In space $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$	In x-y plane $\vec{F} = F_x \hat{i} + F_y \hat{j}$
In space $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$	In x-y plane $\vec{a} = a_x \hat{i} + a_y \hat{j}$
In space $\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$	In x-y plane $\vec{v} = v_x \hat{i} + v_y \hat{j}$
In space $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$	In x-y plane $\vec{r} = x \hat{i} + y \hat{j}$

Ex. If $x = u_x t + \frac{1}{2} a_x t^2$; $y = u_y t + \frac{1}{2} a_y t^2$; $z = u_z t + \frac{1}{2} a_z t^2$ then $\vec{r} = x \hat{i} + y \hat{j} + z \hat{k} = \vec{u}t + \frac{1}{2} \vec{a}t^2$

Ex. The coordinates of a particle moving in YZ plane are given by $y = 4 \cos 6t$ and $z = 6 \sin 6t$. Find :
 (i) The equation of the path (ii) Velocity (iii) Acceleration (iv) The relation between \vec{a} and \vec{r}

Sol. (i) $y = 4 \cos 6t$ & $z = 6 \sin 6t \Rightarrow \frac{y^2}{16} + \frac{z^2}{36} = 1$ (equation of ellipse)

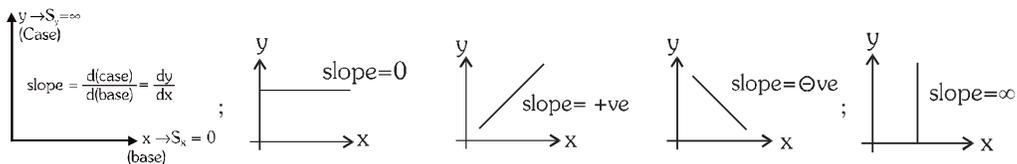
(ii) $\vec{v} = \frac{d\vec{r}}{dt} = \frac{d(y\hat{j} + z\hat{k})}{dt} = \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} = -24 \sin 6t \hat{j} + 36 \cos 6t \hat{k}$

(iii) $\vec{a} = \frac{d\vec{v}}{dt} = -144 \cos 6t \hat{j} - 216 \sin 6t \hat{k}$

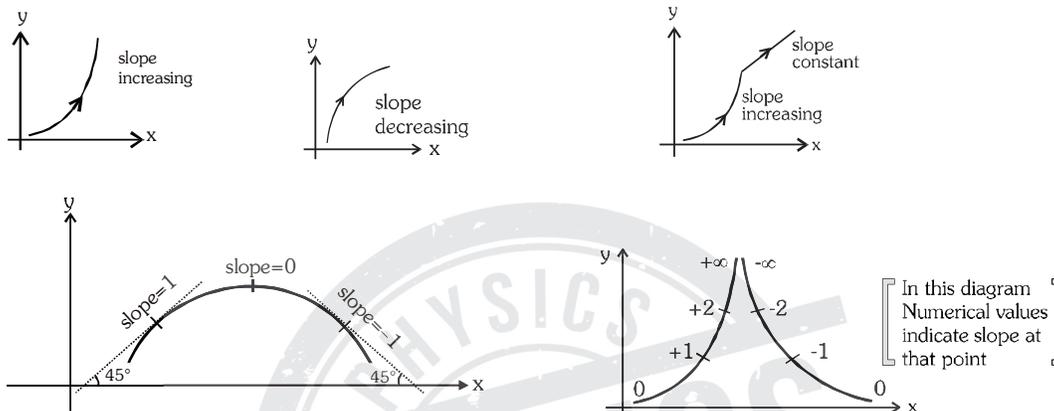
(iv) $\vec{a} = -144 \cos 6t \hat{j} - 216 \sin 6t \hat{k} = -36(4 \cos 6t \hat{j} + 6 \sin 6t \hat{k}) = -36\vec{r} \Rightarrow \vec{a} = -36\vec{r}$

Graph से Related Questions के लिए

Nature of slope(S)



Variation of slope(S)



Relation \rightarrow Function $[y=f(x)] \rightarrow$ Graph का खींचना \rightarrow दिमाग में Slope (dy/dx) आना \rightarrow physical quantity से मिलना

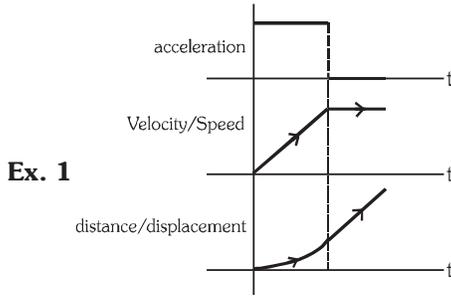
In Kinematics

- Acceleration-t
- Velocity-t
- Speed-t
- Displacement-t
- Distance-t

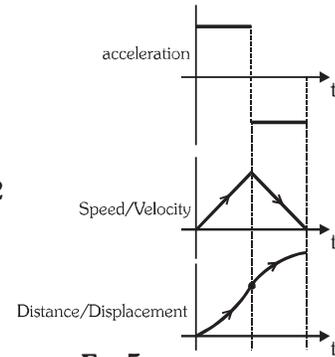
In Mechanics

If on X-axis	& on Y-axis	Then Slope	Formula	$\bar{X}_{av} = \frac{\bar{X}_f - \bar{X}_i}{\text{time}}$
Time (समय)	Displacement (विस्थापन)	Velocity (वेग)	$\bar{v} = \frac{d\bar{s}}{dt}$	$\bar{v}_{av} = \frac{\bar{s}_f - \bar{s}_i}{\text{time}}$
Time (समय)	Velocity (वेग)	Acceleration (त्वरण)	$\bar{a} = \frac{d\bar{v}}{dt}$	$\bar{a}_{av} = \frac{\bar{v}_f - \bar{v}_i}{\text{time}}$
Time (समय)	Momentum (संवेग)	Force (बल)	$\bar{F} = \frac{d\bar{p}}{dt}$	$\bar{F}_{av} = \frac{\bar{p}_f - \bar{p}_i}{\text{time}}$
Time (समय)	Energy (ऊर्जा)	Power (शक्ति)	$P = \frac{dE}{dt}$	$\bar{P}_{av} = \frac{E}{\text{time}}$
	Work (कार्य)	Power (शक्ति)	$P = \frac{dW}{dt}$	$P_{av} = \frac{W}{\text{time}}$
Time (समय)	Angular Momentum (कोणीय संवेग)	Torque (बलाघूर्ण)	$\bar{\tau} = \frac{d\bar{J}}{dt}$	$\bar{\tau}_{av} = \frac{\bar{J}_f - \bar{J}_i}{\text{time}}$

$$a = \frac{dv}{dt} \quad F = \frac{dp}{dt} \quad \Delta v = \int a dt \quad \Delta x = \int v dt \quad I = \Delta p = \int F dt$$



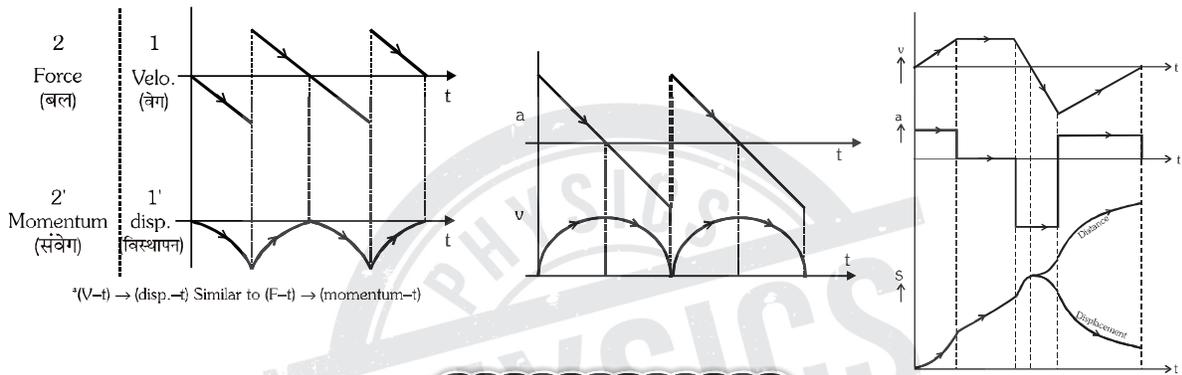
Ex. 2



Ex. 3

Ex. 4

Ex. 5



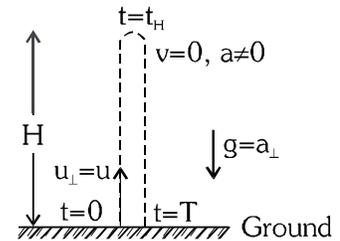
Projectile Motion

■ Upward Vertical Projectile Motion (1-D)

$$t_H = \frac{u}{g} = \frac{u_{\perp}}{a_{\perp}}$$

$$T = 2t_H = \frac{2u}{g} = \text{---}$$

$$H = \frac{u^2}{2g} = \frac{(u_{\perp})^2}{2g}$$



• Motion under gravity (MUG)

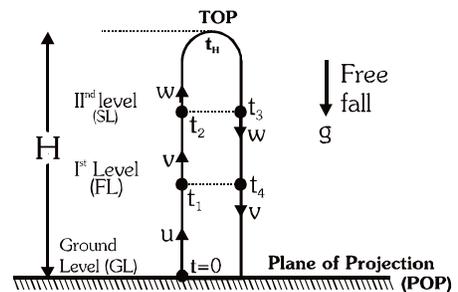
$$GL \rightarrow T = \text{---}$$

$$\frac{2v}{g}$$

$$\frac{2w}{g}$$

Time of journey or total time of flight

$$T = t_1 + t_4 = t_2 + t_3$$

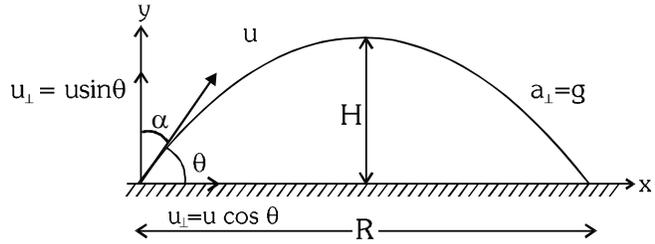


■ **Projectile Motion (2-D)**

$$t_H = \frac{u_{\perp}}{a_{\perp}} = \frac{u \sin \theta}{g} = \frac{u \cos \alpha}{g}$$

$$T = 2t_H = \frac{2u_{\perp}}{a_{\perp}} = \frac{2u \sin \theta}{g} = \frac{2u \cos \alpha}{g}$$

$$H = \frac{u_{\perp}^2}{2a_{\perp}} = \frac{u^2 \sin^2 \theta}{2g} = \frac{u^2 \cos^2 \alpha}{2g}$$



$$* R = (u \cos \theta)T = u_{\parallel} \times \left(\frac{2u_{\perp}}{a_{\perp}} \right) = u_x \times \left(\frac{2u_y}{a_y} \right) = \frac{2u_x u_y}{g}$$

- Maximum Range (अधिकतम परास) $u_x = u_y$ or $\theta = 45^\circ$ $R_{\max} = \frac{u^2}{g}$
- Max. range सदैव acceleration vector की line एवं Range के बीच angle का bisector (अर्द्धक) पर होती है।
- एकसमान चाल (speed) से फेंके गये दो projectiles की Range समान होगी यदि उनके projection angle एक दूसरे के complementary हो अर्थात् एक θ तो दूसरा $90^\circ - \theta$ होता है। Angle $45^\circ + \theta$ एवं $45^\circ - \theta$ पर horizontal range same होती है।

■ **Projectile motion on inclined plane (2-D) - Up motion**

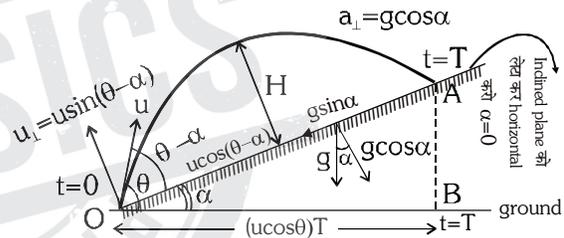
$$t_H = \frac{u_{\perp}}{a_{\perp}} = \frac{u \sin(\theta - \alpha)}{g \cos \alpha}; T = 2t_H = \frac{2u_{\perp}}{a_{\perp}} = \frac{2u \sin(\theta - \alpha)}{g \cos \alpha}; H = \frac{u_{\perp}^2}{2a_{\perp}} = \frac{u^2 \sin^2(\theta - \alpha)}{2g \cos \alpha}$$

* OA = Range on inclined plane

Here OB = OA cos α = (u cos θ)T

$$\Rightarrow OA = \frac{(u \cos \theta)T}{\cos \alpha} = \frac{2u^2 \cos \theta \sin(\theta - \alpha)}{g \cos^2 \alpha}$$

For maximum Range $\theta = \frac{\pi}{4} + \frac{\alpha}{2}$, $R_{\max} = \frac{u^2}{g(1 + \sin \alpha)}$



■ **Projectile motion on inclined plane (2-D) - Down motion (α की जगह - α रखें)**

$$t_H = \frac{u_{\perp}}{a_{\perp}} = \frac{u \sin(\theta + \alpha)}{g \cos \alpha}$$

$$T = 2t_H = \frac{2u_{\perp}}{a_{\perp}} = \frac{2u \sin(\theta + \alpha)}{g \cos \alpha}$$

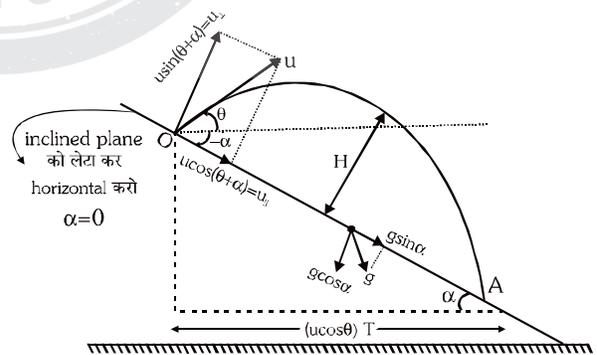
$$H = \frac{u_{\perp}^2}{2a_{\perp}} = \frac{u^2 \sin^2(\theta + \alpha)}{2g \cos \alpha}$$

OA = Range on inclined plane

$$R' = u_{\parallel}T + \frac{1}{2}g_{\parallel}T^2 = \frac{2u^2 \cos \theta \sin(\theta + \alpha)}{g \cos^2 \alpha}$$

or $R \cos \alpha = (u \cos \theta)T \Rightarrow \boxed{R = \frac{(u \cos \theta)}{\cos \alpha} T}$ if $\theta = 0 \Rightarrow \boxed{R = \frac{uT}{\cos \alpha}}$

For maximum range $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$, $R_{\min} = \frac{u^2}{g(1 - \sin \alpha)}$



Special Note : Incline plane पर ऊपर की तरफ motion के सभी सूत्रों (formulae) में α की जगह $-\alpha$ रखने पर नीचे की तरफ motion के सभी सूत्र प्राप्त हो जाते हैं।

Ex. A football is kicked with a velocity of 20m/s at an angle of 45° with the horizontal. [g = 10 m/s²]

(एक फुटबाल क्षैतिज से 45° कोण के साथ 20 m/s के वेग से उछाली जाती है।)

(a) Find the time taken by the football to strike the ground.

(फुटबाल को जमीन तक पहुँचने में लगा समय ज्ञात कीजिए)

(b) Find the maximum height it reaches. (इसकी अधिकतम ऊँचाई ज्ञात कीजिए)

(c) How far away from the kick does it hit the ground ?

(उछालने से कितनी दूरी पर यह जमीन को स्पर्श करेगी)

Sol. (a) We have $u_x = u \cos \theta = 20 \times \cos 45^\circ = 10\sqrt{2} \text{ m/s}$

$$u_y = u \sin \theta = 20 \times \sin 45^\circ = 10\sqrt{2} \text{ m/s}$$

Here $a_x = 0$ and $a_y = -g$

When the football reaches the ground $y = 0$

(जब फुटबाल जमीन पर पहुँचती है।)

$$y = u_y t - \frac{1}{2} g t^2 \Rightarrow 0 = 10\sqrt{2} t - \frac{1}{2} \times 10 \times t^2 \Rightarrow t = 2\sqrt{2} = 2.8 \text{ sec}$$

$$[\text{Direct } T = \frac{2u \sin \theta}{g} = \frac{2 \times 20}{10} \times \frac{1}{\sqrt{2}} = 2.8 \text{ sec}]$$

(b) At maximum height vertical component of velocity becomes zero.

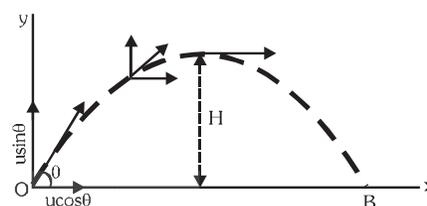
(अधिकतम ऊँचाई पर वेग का ऊर्ध्वाधर घटक शून्य होगा)

$$v_y^2 = u_y^2 - 2gy \Rightarrow 0 = (10\sqrt{2})^2 - 2 \times 10 \times y \Rightarrow y = H = \frac{100 \times 2}{20} = 10 \text{ m}$$

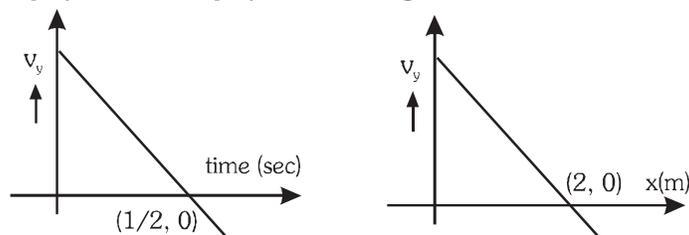
$$[\text{Direct } H = \frac{u^2 \sin^2 \theta}{2g} = \frac{20 \times 20 \times 1}{2 \times 10 \times (\sqrt{2})^2} = 10 \text{ m}]$$

(c) Horizontal distance (क्षैतिज दूरी) [Range (परास)] : $x = R = u_x T = 10\sqrt{2} \times 2\sqrt{2} = 40 \text{ m}$

$$[\text{Direct } R = \frac{u^2 \sin 2\theta}{g} = \frac{20 \times 20}{10} = 40 \text{ m}]$$

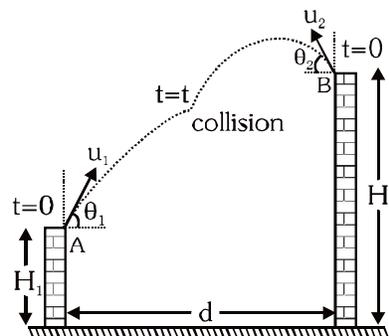


Ex. Two graphs of same projectile motion projected from origin at $t = 0$ are shown. Find u_x & u_y (Take $g = 10 \text{ m/s}^2$)



Ans. Here $\frac{u_y}{g} = \frac{1}{2}$ & $\frac{R}{2} = 2 = \frac{u_x u_y}{g}$ Therefore $u_y = 5 \text{ m/s}^{-1}$ & $u_x = 4 \text{ m/s}^{-1}$

Ex. Two projectiles A and B are projected simultaneously in a vertical plane as shown in figure. Write down necessary conditions for collision to take place.



Sol. For relative motion along horizontal $(u_1 \cos \theta_1 + u_2 \cos \theta_2) t = d$
 For relative motion along vertical $(u_1 \sin \theta_1 - u_2 \sin \theta_2) t = H_2 - H_1$

OR

Relative velocity must be along AB i.e. $\frac{u_1 \sin \theta_1 - u_2 \sin \theta_2}{u_1 \cos \theta_1 + u_2 \cos \theta_2} = \frac{H_2 - H_1}{d}$

Bird-Cage Problem

Case I :

Wire-cage (open cage) : तार का बना पिंजरा [Air of cage is not bound with the cage]
 जब bird (parrot) उड़ती (Flies away) है, तो spring balance की reading decrease होगी।

Note : Bird जब उड़ना प्रारम्भ करती है, तो एक क्षण (for a moment) reading increase होती है।
 इस case में bird चाहे किसी भी तरह उड़े reading decrease होगी।

Case II :

Air tight cage : [Air of cage is bound with the cage]

- जब bird constant velocity से उड़ती है तो spring balance की reading में कोई परिवर्तन नहीं होगा।
- जब bird ऊपर की ओर कुछ acceleration से उड़ना start करती है तो reading increase होगी (या कहे कि पिंजरा भारी लगेगा)



Concept of Pseudo Force

↳ **Not a real force**

↳ **Exist only in non-inertial frame** (Accelerated frames or rotating frames)

Inertial frame (जड़त्विय तन्त्र) → Frames at rest or moving with uniform velocity

Non-Inertial frame (अजड़त्विय तन्त्र) → All the three Newton's laws are not valid.

*Rotating frames में centrifugal force (अपकेन्द्रीय बल) लगता है जो कि Pseudo force है। Pseudo force केवल और केवल observer अर्थात् reference frame के acceleration पर निर्भर करता है। यदि inertial frame है तो pseudo force नहीं लगेगा।

1. A reference frame attached to the earth :

- (a) Is an inertial frame.
- (b) Cannot be an inertial frame because the earth is revolving around the sun.
- (c) Cannot be an inertial frame because the earth is rotating about its axis.
- (d) Is an inertial frame because newton's laws are applicable in this frame.

2. A particle of mass m is observed from an inertial frame of reference and is found to move in a circle of radius r with a uniform speed v . The centrifugal force on it is :-

- (a) $\frac{mv^2}{r}$ towards the centre (b) $\frac{mv^2}{r}$ away from the centre
 (c) $\frac{mv^2}{r}$ along the tangent through the particle (d) Zero

3. A particle of mass m rotates in a circle of radius a with a uniform angular speed ω . It is viewed from a frame rotating with a uniform angular speed ω_0 . The centrifugal force on the particle is:-

- (a) $m\omega^2 a$ (b) $m\omega_0^2 a$ (c) $m\left(\frac{\omega + \omega_0}{2}\right)^2 a$ (d) $m\omega\omega_0 a$

4. A particle of mass m rotates in a circle of radius a with a uniform angular speed ω . It is viewed from a frame rotating with a uniform angular speed ω . The centrifugal force on the particle is

- (a) $m\omega^2 a$ (b) $m\omega_0^2 a$ (c) $m\left(\frac{\omega + \omega_0}{2}\right)^2 a$ (d) $m\omega\omega_0 a$

Answers : 1. b,c 2. d 3. b 4. a

Spring Balance



(नोट : इसकी reading (किलोग्राम-भार) (kg-wt/kg-f) में आती है)

Ex. T_1 T_2 50kg 50kg

Reading = 50 kg-wt = R $\therefore T_1 = T_2 = 50 \times g$

Ex. 1 2 15kg

Reading = 15 kg-wt = R_1
 Reading = 15 kg-wt = R_2

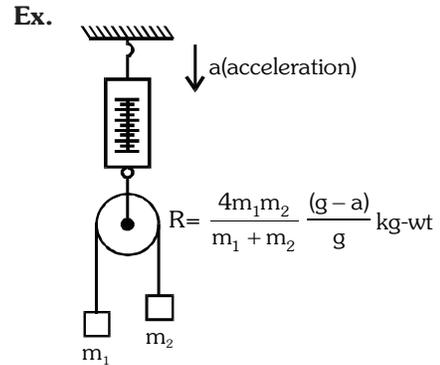
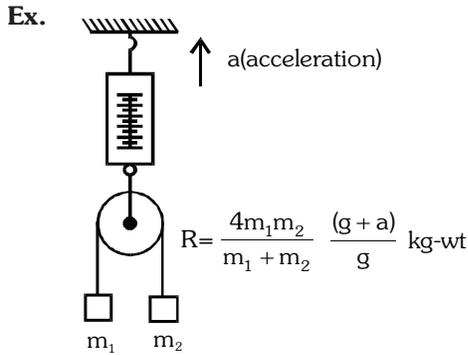
Ex. R = 20 kg-wt

10kg 10kg

Ex. R = $\frac{2T}{g} = \frac{4m_1 m_2}{m_1 + m_2}$

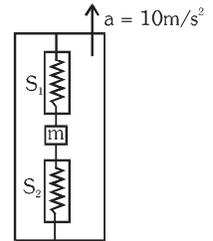
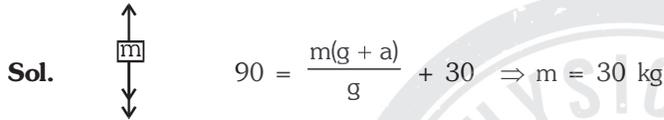
m_1 m_2

Ex. $R_1 = 10$ $R_2 = 10$ 20kg



Ex. Given Reading of $S_1 = 90 \text{ kg}$
Reading of $S_2 = 30 \text{ kg}$ ($g = 10 \text{ m/s}^2$)

If the mass is stationary w.r.t. lift then determine the mass of block.



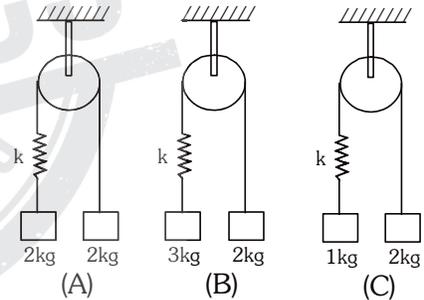
Questions based on Newton's Laws

Ex. If x_1 , x_2 & x_3 be the extensions in the spring in shown cases. Write down mathematical relation between x_1 , x_2 & x_3 .

Ans. $\frac{x_1}{2} = \frac{5x_2}{12} = \frac{3x_3}{4}$

Hint : In (A) $T = kx_1 = 2g$; In (B) : $T = kx_2 = 3g - 3 \times \frac{g}{5} = \frac{12}{5}g$;

In (C) $T = kx_3 = 2g - 2 \times \frac{g}{3} = \frac{4}{3}g$



Ex : एक Monkey (10 kg) 2m/sec^2 के त्वरण से rope पर उपर चढ़ रहा है जब क दूसरा बन्दर (8kg) uniform velocity 2m/sec . से नीचे उतर रहा है। Fixed support पर tension क्या होगा। [रस्सी का mass = 0] [$g = 10\text{ms}^{-2}$]

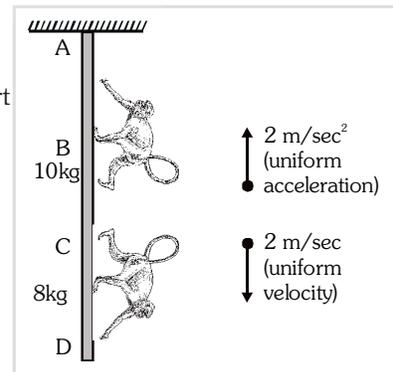
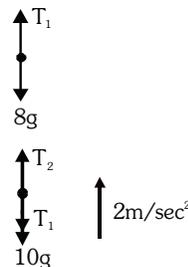
Sol. 8 kg. वाले Monkey के लिए FBD

$$\Rightarrow T_1 = 8g$$

10 kg वाले Monkey के लिए FBD

$$T_2 - 10g - T_1 = 10 \times 2 \quad (\because \vec{F} = m\vec{a})$$

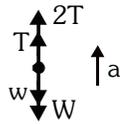
$$\Rightarrow T_2 = 18g + 20 = 200\text{N}$$



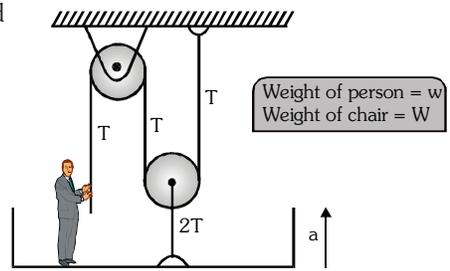
Note : इसमें (rope में) part AB में tension T_2 , Part BC में tension T_1 तथा part CD में tension zero होगा।

Ex. For given situation, determine the force with which the rope is pulled
दी हुई स्थिति के लिए रस्सी को खींचने में कितना बल लगाना पड़ेगा।

Sol. (Chair + Person) का system
लेने पर उनका FBD



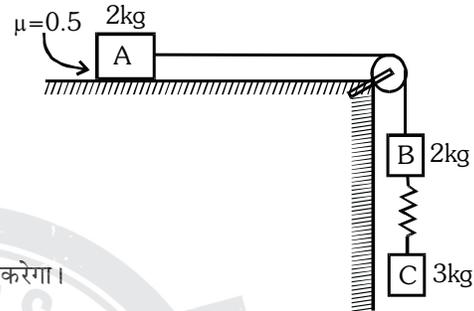
$$3T - w - W = \left(\frac{w + W}{g}\right)a \Rightarrow T = \frac{(w + W)(g + a)}{3g}$$



Ex. दिये गये system में steady state में system का
acceleration ज्ञात कीजिए। {g = 10 ms⁻²}

Sol. $a = \frac{3g + 2g - \mu(2g)}{3 + 2 + 2} = \frac{5g - 0.5 \times 2g}{7} = \frac{4g}{7} = \frac{40}{7} \text{ms}^{-2}$

Note : यहाँ spring force internal force की तरह behave करेगा।



Dependent Motion of Connected Bodies

Method I : Method of constraint equations ($\Sigma x_i = \text{constant}$)

- जितनी moving bodies होगी उतने ही x_1, x_2, \dots होंगे।
- जितनी strings होगी उतनी ही constraint equations बनेगी।

Ex.1 यदि ब्लॉकों के acceleration a_1 एवं a_2 है तो इनके बीच relation ज्ञात कीजिए।

Sol. Here $x_1 + x_2 + x_2 = \text{constant}$

इसको time के साथ differentiate करने पर $\dot{x}_1 + 2\dot{x}_2 = 0$

इसको एक बार और time के साथ differentiate करने पर $\ddot{x}_1 + 2\ddot{x}_2 = 0$

But $\ddot{x}_1 = a_1$ and $\ddot{x}_2 = -a_2$ Therefore $a_1 - 2a_2 = 0 \Rightarrow a_1 = 2a_2$

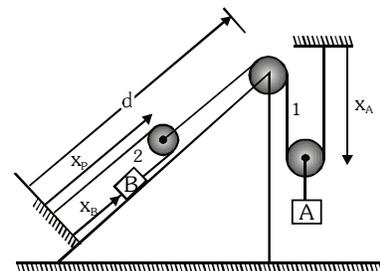
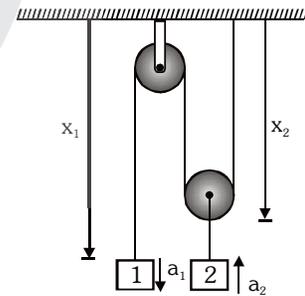
Ex.2 If block A is moving with uniform velocity v_A upwards then find out velocity of block B.

Sol. For string (1) : $2x_A + d - x_P = l_1 \Rightarrow 2\dot{x}_A - \dot{x}_P = 0$

For string (2) $x_P + x_P - x_B = l_2 \Rightarrow 2\dot{x}_P - \dot{x}_B = 0 \Rightarrow 4\dot{x}_A - \dot{x}_B = 0$

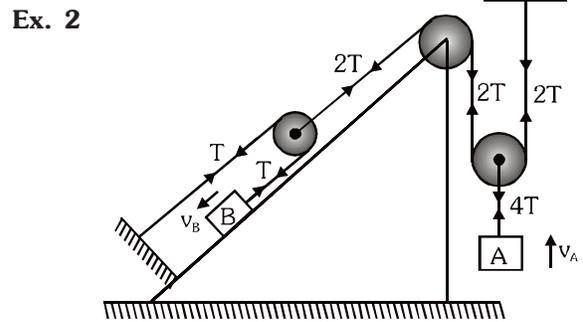
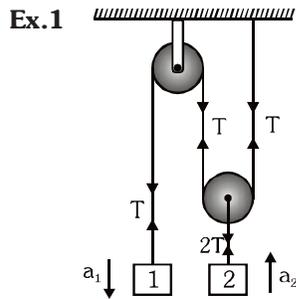
But $\dot{x}_A = -v_A$ so $\dot{x}_B = -4v_A$

Therefore velocity of block B will be $4v_A$ down the incline plane.



Method II : Method of virtual work

(The sum of scalar products of forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero. $\sum \vec{F}_i \cdot \delta \vec{r}_i = 0 \Rightarrow \sum \vec{F}_i \cdot \vec{v}_i = 0 \Rightarrow \sum \vec{F} \cdot \vec{a}_i = 0$)



$$\sum \vec{F}_i \cdot \vec{a}_i = 0 \Rightarrow 2Ta_2 - Ta_1 = 0 \Rightarrow a_1 = 2a_2$$

$$\sum \vec{F}_i \cdot \vec{v}_i = 0 \Rightarrow (4T)v_A - Tv_B = 0 \Rightarrow v_B = 4v_A$$

Friction

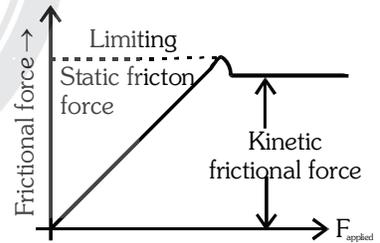
❶ Rough surface [सबसे पहले देखो कि surface rough है या smooth]

❷ Check $\begin{cases} f_r = 0 \\ f_r \neq 0 \end{cases}$

❸ Nature of friction $\begin{cases} f_s \begin{cases} \text{Normal rest} \rightarrow f_r = \text{Applied force} \\ \text{Limiting rest} \rightarrow f_r = \mu_s N \end{cases} \\ f_k \rightarrow f_r = \mu_k N \end{cases}$

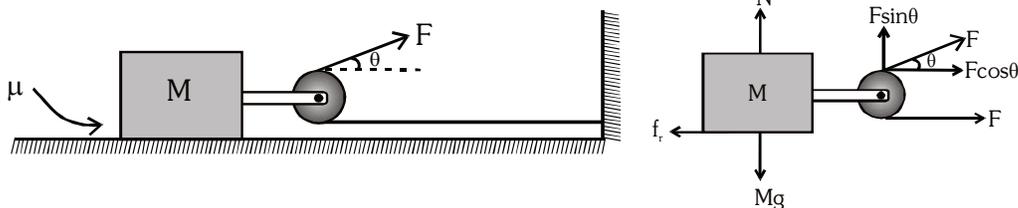
❹ Direction of f_r [f_r = frictional force]

❺ Draw FBD



Ex. $f_r \leq \mu_s Mg$ यदि $F_{\text{applied}} \leq \mu_s Mg$ तो कोई motion नहीं होगा।

Ex. दिए गये system के लिए F की value क्या होगी कि motion start हो जाए।



Note :- Mass M का FBD बनाने के लिए उसको उसके surrounding से अलग (isolate) करना पड़ता है। Earth surface से अलग करने पर N (Normal reaction) उस पर लगाना पड़ेगा। उसको रस्सी (rope) से isolate करने के लिए rope के tension को consider करना पड़ेगा जो कि F के equal है। Pulley को तो mass M से attached माना जा सकता है।

$$N + F\sin\theta = Mg ; \quad F_r = F + F\cos\theta = F(1 + \cos\theta)$$

Motion start करने के लिए $\therefore F_r = \mu N$ अतः $F(1 + \cos\theta) = \mu(Mg - F\sin\theta) \Rightarrow F = \frac{\mu Mg}{1 + \cos\theta + \mu\sin\theta}$

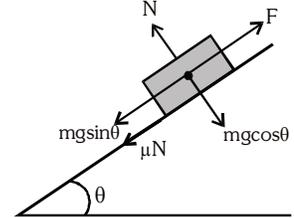
Ex. Body pulled up along a rough inclined plane surface

(एक वस्तु रूक्ष नत समतल सतह पर ऊपर खींची जाती है।)

Normal reaction $N = mg\cos\theta$

$$F_{\min} = mg\sin\theta + \mu mg\cos\theta$$

(Minimum force की condition में speed constant रहेगी)

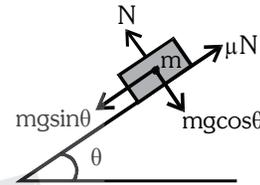


Ex. Body pulled down along a rough inclined plane surface

(वस्तु रूक्ष नत-समतल सतह पर नीचे खींची जाती है।)

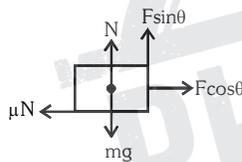
$$F_{\min} = \mu mg\cos\theta - mg\sin\theta$$

(constant velocity से block को नीचे खींचने के लिए F_{\min} से खींचना होगा)



Ex. Block को pull करने के लिए minimum force कितना चाहिए। (चित्र देखें)

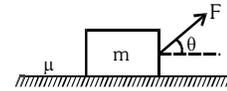
इसका FBD



$$N = mg - F\sin\theta$$

$$F\cos\theta = \mu(mg - F\sin\theta)$$

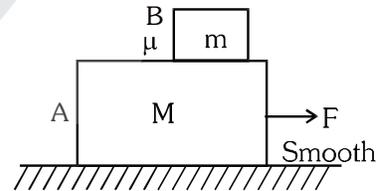
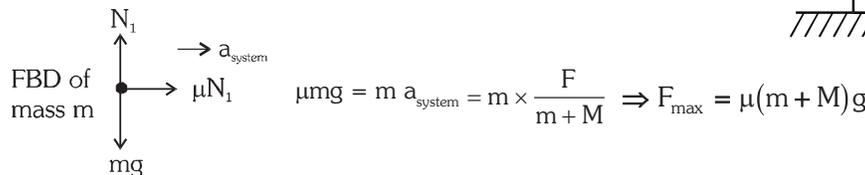
$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$



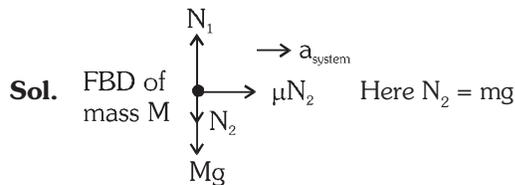
इसके लिये $(\cos\theta + \mu\sin\theta)$ maximum होना चाहिये। इसकी maximum value $\sqrt{1 + \mu^2}$ होती है। अतः $F_{\min} = \frac{\mu mg}{\sqrt{1 + \mu^2}}$

Ex. Find the maximum value of F so that both move together. (There is no relative motion b/w A & B)

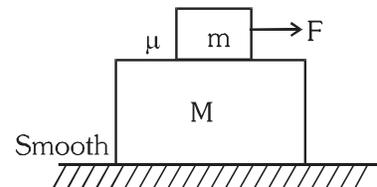
Sol. a_{system} (Acceleration of $m + M$) = $\frac{F}{m + M}$



Ex. Find the maximum value of F so that both move together.



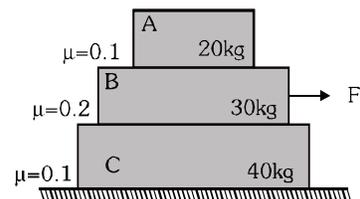
$$a_{\text{system}} = \frac{F}{m + M} = \frac{\mu mg}{M} \Rightarrow F_{\max} = \mu mg \left(1 + \frac{m}{M}\right)$$



Two and Three Blocks Problem in Friction

STEP 1	Decide direction of friction forces	<ul style="list-style-type: none"> The block on which a net force is applied to pull or push, friction on both of its surfaces direction opposite to the force. Friction between two surfaces in contact makes action reaction pair, these forces act in opposite direction with equal magnitude.
STEP 2	Decide range of friction forces between each pair of surfaces in contact.	<ul style="list-style-type: none"> If coefficients of static and kinetic friction are different $f_s \leq \mu_s N$ $f_k = \mu_k N$ If coefficients of static and kinetic friction are same $f \leq \mu N$
STEP 3	Decide maximum possible acceleration each of those blocks, which accelerate only due to friction.	
STEP 4	Sequence of Slipping	<ul style="list-style-type: none"> If friction is zero anywhere below the block on which force is applied, slipping first starts there. Out of the blocks (accelerating due to friction forces only) which has least maximum possible acceleration start slipping next. It starts slipping with respect to that block which has next higher maximum possible acceleration.

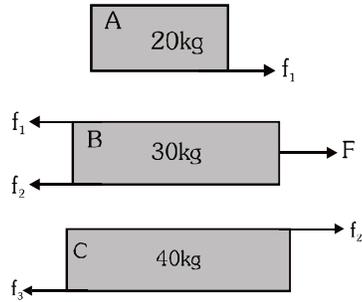
- Ex.** Block B is pulled horizontally by a force F, which increases gradually.
- Find maximum friction between each pair of surface.
 - Decide sequence of sliding between various surfaces in contact. Also calculate minimum value of F for that.
 - Express accelerations of various block as function of F.
 - Draw a-t graph if $F=10t$ where t is time in second. ($g=10 \text{ ms}^{-2}$)



Sol. (i) $f_{1 \max} = (0.1)(20g) = 20\text{N}$

$f_{2 \max} = (0.2)(50g) = 100\text{N}$

$f_{3 \max} = (0.1)(90g) = 90\text{N}$



(ii) Block A and C moves due to frictional forces.

$a_{A \max} = \frac{f_{1 \max}}{m_A} = \frac{20}{20} = 1\text{ms}^{-2}$ & $a_{C \max} = \frac{f_{2 \max}}{m_C} = \frac{100 - 90}{40} = \frac{1}{4}\text{ms}^{-2}$

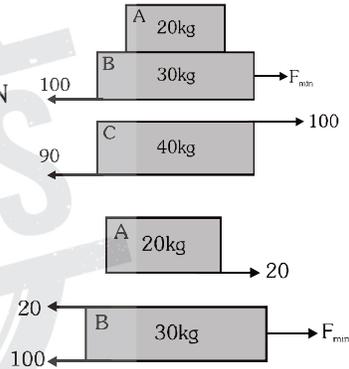
As $a_{C \max} < a_{A \max}$ so slipping between B & C starts before slipping between A & B.
Also as $f_{2 \max} > f_{3 \max}$ so slipping starts first between ground and C.

Calculation of F_{\min}

• Between ground and C : $F_{\min} = 90\text{N}$

• Between B and C : $\frac{F_{\min} - 100}{20 + 30} = \frac{100 - 90}{40} \Rightarrow F_{\min} = 112.5\text{N}$

• Between A and B : $\frac{20}{20} = \frac{F - 20 - 100}{30} \Rightarrow F_{\min} = 150\text{N}$



(iii) For $0 \leq F \leq 90\text{N}$ $a_A = a_B = a_C = 0$

For $90 \leq F \leq 112.5\text{N}$ $a_A = a_B = a_C = \frac{F - 90}{90}$

For $112.5 \leq F \leq 150\text{N}$ $a_A = a_B = \frac{F - 100}{50}, a_C = \frac{100 - 90}{40} = \frac{1}{4}\text{ms}^{-2}$

For $F \geq 150\text{N}$ $a_A = 1\text{ms}^{-2}, a_B = \frac{F - 120}{30}, a_C = \frac{1}{4}\text{ms}^{-2}$

(iv) For $0 \leq t \leq 9\text{s}$ $a_A = a_B = a_C = 0$

For $9 \leq t \leq 11.25\text{s}$ $a_A = a_B = a_C = \frac{t}{9} - 1$

For $11.25 \leq t \leq 15\text{s}$ $a_A = a_B = \frac{t}{5} - 2, a_C = \frac{1}{4}\text{ms}^{-2}$

For $t \geq 15\text{s}$ $a_A = 1\text{ms}^{-2}, a_B = \frac{t}{3} - 4, a_C = \frac{1}{4}\text{ms}^{-2}$

