Matrix

Total Questions: 41

Q1. Question ID: 1365877 | Type: **SC**

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and B = (adjA) and C = 5A then $\frac{|adjB|}{|C|}$ = ...

A. 3

B. 4

C. 1

D. 2

Q2. Question ID: 1545511 | Type: **MP**

If A and B are 3 × 3 matrices and $|A| \neq 0$, then which of the following are true ?

A. $|AB| = 0 \Rightarrow |B| = 0$

B. $|AB| = 0 \Rightarrow B = 0$

C. $|A^{-1}| = |A|^{-1}$

D. $|A + A| = 2|A|$

Q3. Question ID: 1545553 | Type: **MP**

Let $A = a_{ij}$ be a matrix of order 3 where $a_{ij} = \begin{cases} x \text{ if } i = j, x \in R \\ 1 \text{ if } |i - j| = 1 \\ 0 \text{ otherwise} \end{cases}$, then which of the following hold(s) good ?

A. for x = 2, A is a diagonal matrix.

B. A is a symmetric matrix

C. for x = 2, det A has the value equal to 6

D. Let f (x) = det A, then the function f (x) has both the maxima and minima.

Q4. Question ID: 1548297 | Type: **MP**

Let det(adj(adjA)) = 14^4 where A = $\begin{bmatrix} x & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, $x \neq -\frac{11}{3}$, then

A. x = 1

B. det(2A) = 112

C. x = 2

D. det(2A) = 256

Q5. Question ID: 1548340 | Type: **MP**

Let $A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$, then -

A. $7|A| = \frac{1}{2}$

B. $|\text{adj } A| = \frac{1}{196}$

C. $\text{trace (adjA)} = -\frac{1}{7}$

D. Matrix A is a symmetric matrix

Q6. Question ID: 1548363 | Type: **MP**

If $A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$, then which of the following is(are) true ?
(trace of A denotes sum of principal diagonal elements of A)

A. A is invertible

B. trace(adj(adj(A))) = 144

C. trace(adj(adj(A))) = 8

D. |adj A| is less than 400

Q7. Question ID: 1548624 | Type: **IS**

If the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (a, b, c, d not all simultaneously zero) commute, find the value of $\frac{d-b}{a+c-b}$. Also show that the matrix which commutes with A is of the form $\begin{bmatrix} \alpha-\beta & 2\beta/3 \\ \beta & \alpha \end{bmatrix}$

Q8. Question ID: 1548687 | Type: **IS**

If A is an idempotent non-zero matrix and I is an identity matrix of the same order, find the value of n, $n \in N$, such that $(A + I)^n = I + 127 A$.

Q9. Question ID: 1548718 | Type: **IS**

Let $A = \begin{bmatrix} 3x^2 \\ 1 \\ 6x \end{bmatrix}$, $B = [a\ b\ c]$ and $C = \begin{bmatrix} (x+2)^2 & 5x^2 & 2x \\ 5x^2 & 2x & (x+2)^2 \\ 2x & (x+2)^2 & 5x^2 \end{bmatrix}$ be three given matrices, where a, b, c and $x \in R$. Given that tr(AB) = tr(C) $\forall x \in R$, where tr(A) denotes trace of A. Find the value of (a + b + c)

Q10. Question ID: 1549061 | Type: **IM**

Let $A = \begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$ are two matrices such that $AB = (AB)^{-1}$ and $AB \neq I$ (where I is an identity matrix of order 3 × 3).

Find the value of Tr. $\left(AB + (AB)^2 + (AB)^3 + ... + (AB)^{100}\right)$,
where Tr. (A) denotes the trace of matrix A.

Q11. Question ID: 1549163 | Type: **IS**

Let A be a 3 × 3 matrix such that $a_{11} = a_{33} = 2$ and all the other $a_{ij} = 1$. Let $A^{-1} = xA^2 + yA + zI$, then find the value of (x + y + z) where I is a unit matrix of order 3.

Q12. Question ID: 1549319 | Type: **IM**

$A_{3 \times 3}$ is a matrix such that |A|=a, B = (adj A) such that |B|= b. Find the value of $(ab^2 + a^2b + 1)S$ where $\frac{1}{2}S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \ldots\ldots$ up to ∞, and a = 3.

Q13. **Question ID: 1842524** | Type: **SC**

If A and B are symmetric matrices, then ABA is -

A. symmetric matrix

B. skew symmetric matrix

C. diagonal matrix

D. scalar matrix

Q14. **Question ID: 1842651** | Type: **IS**

A is a square matrix of order n.
ℓ = maximum number of distinct entries if A is a triangular matrix
m = maximum number of distinct entries if A is a diagonal matrix
p = minimum number of zeroes if A is a triangular matrix. If $\ell + 5 = p + 2m$, find the order of the matrix.

Q15. **Question ID: 1842658** | Type: **IS**

Let A be the set of all 3 × 3 skew symmetric matrices whose entries are either –1, 0 or 1. If there are exactly three 0's, three 1's and three (–1)'s, then find the number of such matrices.

Q16. **Question ID: 2147833** | Type: **SC**

If A be a square matrix satisfying $A^2 + 5A + 5I = 0$ then the inverse of (A + 3I) is equal to

A. A – 2I

B. A – 3I

C. A + 2I

D. A + I

Q17. **Question ID: 2242308** | Type: **MP**

Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

A. $A^2 - 4A - 5I_3 = 0$

B. $A^{-1} = \frac{1}{5}(A - 4I_3)$

C. A^3 is not invertible

D. A^2 is invertible

Q18. **Question ID: 2393203** | Type: **MP**

Let B is an invertible square matrix and B is the adjoint of matrix A such that $AB = B^T$, then-

A. A is an identity matrix

B. B is a symmetric matrix

C. A is not an identity matrix

D. B is a skew symmetric matrix

Q19. **Question ID: 2393274** | Type: **MP**

If M be a non-singular matrix of order 3 such that $M^{-1} = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \\ 5 & 3 & 4 \end{bmatrix}$, then choose the correct option(s) -

(where |X| represent determinant of matrix X, adj(X) denotes adjoint of matrix X and Tr(X) denotes trace of matrix X)

A. Absolute value of (15tr(adjM)) is 5

B. $|M| = -\frac{1}{36}$

C. $M^{-2} = \begin{bmatrix} 50 & 47 & 47 \\ 47 & 50 & 47 \\ 47 & 47 & 50 \end{bmatrix}$

D. $-36\,\mathrm{adj}\,M = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

Q20. **Question ID: 2599919** | Type: **ID**

Consider a 3 × 3 matrix A = $\{a_{ij}\}$ where $a_{ij} = \frac{1}{i+j-1} \forall i, j \in \{1,2,3\}$. If matrix B = $\{b_{ij}\}$ of order 3 × 3 exists such that A^{-1} = 3B, then the value of $\frac{|B|}{a_{23}.b_{22}}$ is (where |X| denotes determinant value of matrix X)

Q21. **Question ID: 2652356** | Type: **IS**

Let β be a real number. Consider the matrix $A = \begin{pmatrix} \beta & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & -2 \end{pmatrix}$. If $A^{2022} - (\beta - 1)A^{2021} - \beta A^{2020}$ is a singular matrix, then the value of 9β is ________

Q22. **Question ID: 2711783** | Type: **SC**

Let B be a square matrix such that |B| = 1 and $2A + B = B^T - A^T$, then |A + B| is -

A. 0

B. –1

C. 1

D. 2

Q23. **Question ID: 3076** | Type: **MP**

If M be a non-singular matrix of order 3 such that $M^{-1} = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \\ 5 & 3 & 4 \end{bmatrix}$, then choose the correct option(s) - (where |X| represent determinant of matrix X, adj(X) denotes adjoint of matrix X and Tr(X) denotes trace of matrix X)

A. Absolute value of (15tr(adjM)) is 5

B. $|M| = -\frac{1}{36}$

C. $M^{-2} = \begin{bmatrix} 50 & 47 & 47 \\ 47 & 50 & 47 \\ 47 & 47 & 50 \end{bmatrix}$

D. $-36\,\text{adj}\,M = \begin{bmatrix} 3 & 4 & 5 \\ 4 & 5 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

Q24. **Question ID: 3201445** | Type: **SC**

60 gm urea was dissolved in 1L water. Find amount of water vapourised if temperature of the solution is raised to 101.5°C.
[Given K_b (water) = 0.5 k kg mol^{-1}, density of water = 1 gm mL^{-1}]

A. 450 g

B. 633.33 g

C. 650 g

D. 666.67 g

Q25. **Question ID: 3201865** | Type: **SC**

Consider following solutions :
(P) 1M aqueous glucose solution
(Q) 1M aqueous sodium chloride solution
(R) 1M aqueous ammonium phosphate solution
(S) 1M benzoic acid in benzene (α = 1)
Select the INCORRECT statements -

A. $\pi_P = \pi_S$

B. $\pi_R > \pi_S$

C. $\pi_R > \pi_Q$

D. $\pi_Q > \pi_P$

Q26. **Question ID: 3201911** | Type: **SC**

When 20 g of naphtholic acid ($C_{11}H_8O_2$) is dissolved in 50 g of benzene (K_f = 1.72 K kg mol^{-1}), a freezing point depression of 2 K is observed. The Van't Hoff Factor (i) is

A. 0.5

B. 1

C. 2

D. 3

Q27. **Question ID: 3213659** | Type: **MP**

The cyanide process of gold extraction involves leaching out gold from its ore with $CN^{\ominus}$ in the presence of **Q** in water to form **R**. Subsequently, **R** is treated with **T** to obtain Au and **Z**. Choose the correct option(s).

A. **T** is Zn

B. **R** is $[Au(CN)_4]^{\ominus}$

C. **Z** is $[Zn(CN)_4]^{2\ominus}$

D. **Q** is O_2

Q28. **Question ID: 3703463** | Type: **SC**

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ and det. $(A^n - I) = 1 - \lambda^n$, $n \in N$ then the value of λ, is -

A. 1

B. 2

C. 3

D. 4

Q29. **Question ID: 3703489** | Type: **SC**

If an idempotent matrix is also skew symmetric then it must be-

A. an involutary matrix

B. an identity matrix

C. an orthogonal matrix

D. a null matrix

Q30. **Question ID: 3703631** | Type: **SC**

Let $A = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}$ where a and b are real number. If A^2 is a null matrix then the product ab equals-

A. 0

B. 1

C. –1

D. ± 1

Q31. **Question ID: 3703708** | Type: SC

If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$, then -

A. $a = 1, c = -1$

B. $a = 2, c = -\frac{1}{2}$

C. $a = -1, c = 1$

D. $a = \frac{1}{2}, c = \frac{1}{2}$

Q32. **Question ID: 3703753** | Type: SC

If A is skew symmetric matrix of order 3 and X be another matrix of same order, then $|XA + AX^T|$ is (where |P| denotes determinant of matrix P) -

A. $|X + X^T|$

B. $|A + X|$

C. $|A - X|$

D. 0

Q33. **Question ID: 3703815** | Type: SC

If A and B are symmetric matrices of the same order and X = AB + BA, Y = AB – BA then $(XY)^T$ is equal to :

A. XY

B. YX

C. –YX

D. None

Q34. **Question ID: 3703866** | Type: SC

Let $A = \begin{bmatrix} \ell-3 & a & b \\ c & 6 & d \\ e & f & 9-\ell \end{bmatrix}$, B = adj(A) and C = adj(B). If |A| = 5, then tr(C) is (where |X|, tr(X) & adj(X) denote determinant value, trace and adjoint of matrix X respectively) -

A. 5

B. 12

C. 30

D. 60

Q35. **Question ID: 3703908** | Type: **SC**

AB = A and BA = B, then (here A & B are matrix of n × n) which of the following must be true -

A. A = B

B. $A^2 = A$

C. A = I

D. B = I

Q36. **Question ID: 3708412** | Type: **IS**

Let A, B and A + B are non-singular matrices of order 3 × 3 satisfying $A^{-1} + B^{-1} = (A + B)^{-1}$ and $|AB^{-1}| \in R$ then value of $\frac{|A|}{|B|}$ is

Q37. **Question ID: 3708510** | Type: **IS**

Let P be a 2 × 2 matrix such that $P\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $P^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. If p_1 and p_2 ($p_1 > p_2$) are two values of p for which det(P – pI) = 0, where I is an identity matrix of order 2, then ($5p_1 + 2p_2$) is equal to
[**Note :** det(M) denotes determinant of square matrix M]

Q38. **Question ID: 3708576** | Type: **IM**

Number of 3 × 3 symmetric matrices which can be formed by three '0', three '1' & three '–1' only, is

Q39. **Question ID: 3708638** | Type: **IS**

An invertible matrix A of order 3 satisfies the relation $A = A^{-1} + 2I$, (where I denotes identity matrix). The value of $|A - I|.|A + I|.|A - 2I|$ is

Q40. **Question ID: 395084** | Type: **MP**

If $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$ then value(s) of α for which $A^2 \neq B$ is

A. 1

B. –1

C. 4

D. –4

Q41. **Question ID: 4198996** | Type: **SC**

If $A = \begin{bmatrix} e^t & e^{-t}\cos t & e^{-t}\sin t \\ e^t & -e^{-t}\cos t - e^{-t}\sin t & -e^{-t}\sin t + e^{-t}\cos t \\ e^t & 2e^{-t}\sin t & -2e^{-t}\cos t \end{bmatrix}$. Then A is -

A. Invertible only if $t = \frac{\pi}{2}$

B. not invertible for any t ε R

C. invertible for all t ε R

D. invertible only if t = π

Solutions

Q1. Answer: C

Solution: $\frac{|adjB|}{|C|} = \frac{|adj(adjA)|}{|5A|} = \frac{|A|^{(3-1)^2}}{5^3|A|} = \frac{|A|^3}{5^3}$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{vmatrix} = 5$$

$\Rightarrow \frac{|A|^3}{5^3} = 1$

Q2. Answer: N/A

Solution: $|AB| = 0 \Rightarrow |A|.|B| = 0 \Rightarrow |A| = 0$ or $|B| = 0 \Rightarrow$ option (A)

$|A^{-1}| = |A|^{-1} \Rightarrow$ option (C) option D : $|A + A| = |2A| = 2^3|A|$ $\because (|kA| = k^n|A|)$

Q3. Answer: N/A

Solution: $A = \begin{bmatrix} x & 1 & 0 \\ 1 & x & 1 \\ 0 & 1 & x \end{bmatrix}$

$\det(A) = x(x^2 - 1) - 1(x)$

$f(x) = x^3 - x - x = x^3 - 2x$

$f(2) = 8 - 4 = 4$

$f'(x) = 3x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{\frac{2}{3}}$

$\therefore$ f(x) has both maxima & minima

$\Rightarrow$ option (B) & (D)

Q4. Answer: N/A

Solution: $|\text{adj}(\text{adj}A)| = 14^4$

$|A|^4 = 14^4$

$\begin{vmatrix} x & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 14^4$

$\Rightarrow x = 1$ and $\det(2A) = 112$

Q5. Answer: N/A

Solution: $A^{-1} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

$|A^{-1}| = -14$

$|A| = -\frac{1}{14}$

$|\text{adj}A| = |A|^2 = \frac{1}{196}$

adj A = $|A|\, A^{-1}$

$\Rightarrow$ tr (adj A) = $|A| + 2|A| - |A| = -\frac{1}{7}$

$\Rightarrow A^{-1}$ is symmetic $\Rightarrow$ A is symmetric

Q6. Answer: N/A

Solution: $A = \begin{bmatrix} 3 & 0 & 0 \\ 2 & 2 & 0 \\ 4 & 5 & 3 \end{bmatrix}$

$|A| = 18 \Rightarrow$ A is invertible

tr (adj(adjA))

tr $(|A|\, A) = 3|A| + 2|A| + 3|A|$

$= 8|A| = 144$

$|\text{adj}A| = |A|^2 = 18^2 < 400$

Q7. Answer: N/A

Solution: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

given AB = BA

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4a \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$$

$a + 2c = a + 3t$ ...(i) $\quad c = \frac{3b}{2}$

$b + 2d = a + 4b$ ...(ii) $\quad a = 2d - 3b$

$3a + 4c = c + 3d$ ...(iii)

$3b + 4d = 2c + 4d$ ...(iv) $\quad c = \frac{3b}{2}$

put $\frac{d-b}{a+c-b}$

$$\frac{a-b}{\frac{2a-3b}{2}+\frac{3b}{2}-b} = \frac{2(d-b)}{2d-3b+3b-2b} = \frac{2(d-b)}{2(d-b)} = 1$$

put $d = \alpha$

$c = \beta$

$c = \frac{3b}{2}$

$b = \frac{3c}{3} = \frac{2\beta}{3}$

$a = \frac{2d-3b}{2} = \frac{2\alpha - 3\times\frac{2\beta}{3}}{2} = \alpha - \beta = 0$

Q8. Answer: N/A

Solution: $(A + I)^n = I + 127A$ ....(1)

$(A + I)^2 = I + 3A = I + (2^2 - 1)A$

$(A + I)^3 = I + 7A = I + (2^3 - 1)A$

$(A + I) = I + 15A = I + (2^4 - 1)A$

⋮

$(A + I)^n = I + (2^n-1)A \Rightarrow 2^n-1 = 127 \Rightarrow n = 7$

Q9. Answer: N/A

Solution: tr (AB) = tr(C)

$3ax^2 + b + 6cx = (x + 2)^2 + 2x + 5x^2 \quad \forall x \in R$

$\Rightarrow a = 2, b = 4, c = 1 \Rightarrow a + b + c = 7$

Q10. Answer: N/A

Solution: $AB = \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix}$

$(AB) = (AB)^{-1} \Rightarrow (AB)^2 = I \Rightarrow x = -1/5$

$tr(AB + (AB)^2 + (AB)^3 + \ldots\ldots + (AB)100) = tr(AB + I + AB + I \ldots\ldots + I) = tr(50AB + 50I)$

$= tr(50AB + 50I) = 50(10x + 1) + 50(3) = 50(-1) + 150 = 100$

Q11. Answer: N/A

Solution: $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$A^{-1} = xA^2 + yA + zI$

$xA^3 + yA^2 + zA - I = 0$ ....(1)

Charactristic equation of A

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$(2 - \lambda)((1 - \lambda)(2 - \lambda) - 1((2 - \lambda) - (1)) + 1(1-(1-\lambda))$

$(2 - \lambda)(1 - I)(2 - \lambda) - 2 + \lambda - 1 + \lambda + \lambda = 0$

$(\lambda - 1)(\lambda - 2)2 - 3 + 3\lambda = 0$

$(\lambda - 1)(\lambda^2 - 4\lambda + 7) = 0$ ......(2)

Comparing (1) and (2)

$x = \frac{1}{7}$, $y = \frac{-5}{7}$, $z = \frac{11}{7}$

$x + y + z = 1$

Q12. Answer: N/A

Solution: $\frac{1}{2}S = \frac{a}{b} + \frac{a^2}{b^3} + \frac{a^3}{b^5} + \ldots + \infty = \frac{\frac{a}{b}}{1 - \frac{a}{b^2}} = \frac{a}{b\left(\frac{b^2 - a^2}{b^2}\right)} = \frac{ab}{b^2 - a} = \frac{3b}{b^2 - 3}$

$|B| = b \Rightarrow |\text{adj } A| = b$

$a^2 = b \Rightarrow b = 9$

$(ab^2 + a^2b + 1)S = 225$

Q13. Answer: A

Solution: Given $A^T = A$

$B^T = B$

As 'A' and 'B' are symmetric matrices ...(1)

$\Rightarrow (ABA)^T = ((AB)A)^T$

$\Rightarrow A^T.(AB)^T$

$\Rightarrow A^T.B^T.A^T$

$\Rightarrow ABA$ from Eq. (1)

$\therefore$ ABA is symmetric matrix

Q14. Answer: N/A

Solution: $\ell = \frac{n(n+1)}{2} + 1$, $m = n + 1$, $p = \frac{n(n-1)}{2}$

$\therefore \frac{n(n+1)}{2} + 6 = \frac{n(n-1)}{2} + 2(n+1)$

$\Rightarrow n^2 + n + 12 = n^2 - n + 4n + 4$

$\Rightarrow 8 = 2n \Rightarrow n = 4$

$\therefore$ order of matrix = 4

Q15. Answer: N/A

Solution: In a skew symmetric matrix, diagonal elements are zero.

$$\begin{bmatrix} 0 & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{bmatrix}$$

Also $a_{ij} + a_{ji} = 0$

Hence number of matrices = 2 × 2 × 2 = 8

Q16. Answer: C

Solution:

Q17. Answer: N/A

Solution: $A^2 - 4A - 5I_3 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$-4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore A^2 - 4A - 5I_3 = 0$

or $A^{-1} A^2 - 4A^{-1} A - 5A^{-1} I_3 = 0$ or $(A^{-1} A) A - 4I_3 - 5A^{-1} = 0$ or $IA - 4I_3 - 5A^{-1} = 0$

$\therefore A^{-1} = (A - 4I_3)$

Also, $|A^2| = \begin{vmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{vmatrix} = 9\,(81 - 64) - 8(72 - 64) + 8(64 - 72)$

$= 9 \times 17 - 8 \times 8 + 8 \times (-8) = 133 - 128 = 5 \neq 0$

$\therefore A^2$ is invertible

and $A^3 = A \,.\, A^2 = A.(4A - 5I_3) = 4A^2 - 5A$

$$= \begin{bmatrix} 36 & 32 & 32 \\ 32 & 36 & 32 \\ 32 & 32 & 36 \end{bmatrix} + \begin{bmatrix} -5 & -10 & -10 \\ -10 & -5 & -10 \\ -10 & -10 & -5 \end{bmatrix} = \begin{bmatrix} 31 & 22 & 22 \\ 22 & 31 & 22 \\ 22 & 22 & 31 \end{bmatrix}$$

$\therefore |A^3| \neq 0$

$\therefore A^3$ is invertible.

Q18. Answer: N/A

Solution: $AB = B^T$

$\Rightarrow A = B^T B^{-1}$

$|A| = |B^T|. |B^{-1}| = 1$ ...(1)

Also $\text{Adj } A = \text{Adj}(B^T B^{-1})$

$= \text{Adj}(B^{-1})\,\text{Adj}(B^T)$

$\text{Adj}A = (\text{Adj}B)^{-1}\,\text{Adj}(B^T)$

$\Rightarrow (\text{Adj}B)^T = (\text{Adj}B)\,\text{Adj}A$

$= (\text{Adj}B)\,B$

$\Rightarrow (\text{Adj}B)^T = |B|.I$

$\Rightarrow$ AdjB = |B|.I

$\Rightarrow$ Adj(AdjA) = |AdjA| I

$\therefore |A|^{n-2}A = |A|^{n-1}I$ (order n)

$\Rightarrow$ A = |A|I $\Rightarrow$ A = I (By (1))

$\therefore$ B = I

Q19. Answer: N/A

Solution: $|M^{-1}| = \frac{1}{|M|} = -36 \Rightarrow |M| = -\frac{1}{36}$

$\therefore \text{adj}M = |M|M^{-1} = -\frac{1}{36}M^{-1} \Rightarrow -36\text{adj}M = M^{-1}$

& $\text{Tr}(\text{adj}M) = -\frac{1}{36}(3+5+4) = -\frac{1}{3}$

$\therefore$ Absolute value of (15tr(adjM)) = 5

& $M^{-2} = M^{-1}.M^{-1} = \begin{bmatrix} 50 & 47 & 47 \\ 47 & 50 & 47 \\ 47 & 47 & 50 \end{bmatrix}$

Q20. Answer: N/A

Solution: $A = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$

$\Rightarrow B = \frac{1}{3}A^{-1} = \frac{1}{3}\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -120 & 180 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} 3 & -12 & 10 \\ -12 & 64 & -60 \\ 10 & -60 & 60 \end{bmatrix}$

Q21. Answer: N/A

Solution: |A| ≠ 0

$|A^{2020}(A^2 - (\beta - 1)A - \beta I)| = 0$

$\Rightarrow |A^{2020}(A^2 + A - \beta(A + I))| = 0$

$\Rightarrow |A^{2020}(A(A + I) - \beta(A + I))| = 0$

$\Rightarrow |A^{2020}(A - \beta I)(A + I)| = 0$

$|A^{2020}| \neq 0, |A + I| \neq 0$

$\Rightarrow |A - \beta I| = 0 \Rightarrow \beta = \frac{1}{3}$

Q22. Answer: C

Solution: $2A + B = B^T - A^T$ ...(i)

take transpose both side

$2A^T + B^T = B - A$ ...(ii)

equation (i) – equation (ii)

$\Rightarrow 3A = -3A^T \Rightarrow A^T = -A$

$\Rightarrow 2A + B = B^T + A$

$\Rightarrow A + B = B^T$

$\Rightarrow |A + B| = |B^T| = |B| = 1$

Q23. Answer: N/A

Solution: $|M^{-1}| = \frac{1}{|M|} = -36 \Rightarrow |M| = -\frac{1}{36}$

$\therefore$ adjM = $|M|M^{-1} = -\frac{1}{36}M^{-1} \Rightarrow -36\text{adj}M = M^{-1}$

& Tr(adjM) = $-\frac{1}{36}(3+5+4) = -\frac{1}{3}$

$\therefore$ Absolute value of (15tr(adjM)) = 5

& $M^{-2} = M^{-1}.M^{-1} = \begin{bmatrix} 50 & 47 & 47 \\ 47 & 50 & 47 \\ 47 & 47 & 50 \end{bmatrix}$

Q24. Answer: D

Solution: $\Delta T_b = K_b \times m$

$1.5 = 0.5 \times \frac{60}{60} \times \frac{100}{\text{wt. of solvent}}$

wt. of solvent = 333.33g

So, water present in vapour phase =1000 – 333.33 = 666.67 g

Q25. Answer: A

Solution:

π = iCRT

For glucose, i = 1;	$\pi_P = RT$
NaCl, i = 2;	$\pi_Q = 2RT$
$(NH_4)_3PO_4$, i = 4;	$\pi_R = 4RT$
Benzoic acid, i = 2;	$\pi_S = 2RT$

Q26. Answer: A

Solution: $\Delta T_f = i \times K_f \times m$

$2 = i \times 1.72 \times \frac{\frac{20}{M_{solute}}}{\frac{50}{1000}}$

$2 = i \times 1.72 \times \frac{20}{172} \times \frac{1000}{50}$

$i = \frac{1}{2}$

Q27. Answer: N/A

Solution: $4Au(s) + 8CN^{\ominus}(aq) + 2H_2O(aq) + O_2(g) \rightarrow 4[Au(CN)_2]^{\ominus}(aq) + 4OH^{\ominus}(aq)$

(Q)

$2[Au(CN)_2]^{\ominus}(aq) + Zn(s) \rightarrow [Zn(CN)_4]^{2\ominus}(aq) + 2Au(s)$

(R) (T) (Z)

Q28. Answer: B

Solution: $\because A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

$\Rightarrow A^3 = A^2 \cdot A = 2A^2 = 2^2A$

Similarly , $A_n = 2^{n-1}A$

$\therefore A^n - I = \begin{bmatrix} 2^{n-1} & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 2^{n-1}-1 & 2^{n-1} \\ 2^{n-1} & 2^{n-1} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $\det(A^n - I) = (2^{n-1} - 1)^2 - (2^{n-1})^2$

$=$ $1 - 2^n = 1 - \lambda^n$ (given)

$\therefore$ $\lambda = 2$

Q29. Answer: D

Solution: $A' = -A$ and $A^2 = A$

So $(A^2)' = A'$ Þ $(A')^2 = -A$

$\Rightarrow$ $(-A)^2 = -A$ $\Rightarrow A = -A$

$\Rightarrow$ $A = 0$

Q30. Answer: C

Solution: $A = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix} \begin{bmatrix} a & 1 \\ -1 & b \end{bmatrix}$$

$$= \begin{bmatrix} a^2-1 & a+b \\ -(a+b) & b^2-1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a^2 - 1 = 0$, $b^2 - 1 = 0$

and $a + b = 0$

$\Rightarrow$ $ab = -1$

Q31. Answer: A

Solution: Given

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} \; ; \quad A^{-1} = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$$

We know,

$AA^{-1} = I$

$$\Rightarrow \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & a & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ -4 & 3 & c \\ 5/2 & -3/2 & 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4+5 & 3-3 & c+1 \\ 1/2 - 8 + \frac{15}{2} & 1/2 + 6 - \frac{9}{2} & 1/2 + 2c + \frac{3}{2} \\ 3/2 + (-4a) + 5/2 & 3/2 + 3a - \frac{3}{2} & 3/2 + ac + 1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & c+1 \\ 0 & 6 - \frac{8}{2} & 2c+2 \\ 4-4a & 3a & ac+2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Comparing each entry of the matrices on both sides, we get

$c + 1 = 0$

$\Rightarrow$ $c = -1$

and $4 - 4a = 0$

$\Rightarrow$ $4a = 4$

$\Rightarrow$ a = 1

$\therefore$ a = 1, c = – 1

Q32. Answer: D

Solution: $A^T = -A$

Let $B = XA + AX^T$

$B^T = (XA + AX^T)^T = (XA)^T + (AX^T)^T$

$= A^T X^T + XA^T$

$= -AX^T - XA$

$= -B$

$\therefore$ B is also skew symmetric matrix of order 3.

$\therefore$ $|B| = |XA + AX^T| = 0$

Q33. Answer: C

Solution: Given that A and B are symmetric.

$A^T = A, B^T = B$

X = AB + BA

$\Rightarrow X^T = (AB)^T + (BA)^T$

$= B^T A^T + A^T B^T = BA + AB = X, \quad ((AB)^T = B^T A^T)$

Y = AB – BA

$\Rightarrow Y^T = (AB)^T - (BA)^T$

$= B^T A^T - A^T B^T = BA - AB = -Y, \quad ((AB)^T = B^T A^T)$

Now $(XY)^T = Y^T X^T = -YX$

Q34. Answer: D

Solution: B = adj (A)

C = adj (B) = adj (adj A)

$C = |A|^{n-2} A$

= 5A

tr(C) = 5tr(A) = 5 × 12 = 60

Q35. Answer: B

Solution: Given, AB = A and BA = B

Take, AB = A

Þ A(BA) = A [Since, B = BA]

Þ (AB)A = A

Þ AA = A [Since, AB = A]

Þ $A^2 = A$

Q36. Answer: N/A

Solution: $A^{-1} + B^{-1} = (A + B)^{-1}$

$\left(A^{-1} + B^{-1}\right) = (A + B) = I$

$I + A^{-1}B + B^{-1}A + I = I$

$I + A^{-1}B + B^{-1}A = 0$

$I + A^{-1}B + \left(A^{-1}B\right)^{-1} = 0$

$\left(A^{-1}B\right) + \left(A^{-1}B\right)^{-1} = -I$

$\left(A^{-1}B\right) + I = -\left(A^{-1}B\right)$

$\left(A^{-1}B\right)^2 + \left(A^{-1}B\right) + I = 0$

$\left(A^{-1}B - I\right)\left(\left(A^{-1}B\right)^2 A^{-1}B + I\right) = 0$

$\left(A^{-1}B\right)^3 = I \rightarrow \left|A^{-1}B\right| = 1$

Q37. Answer: N/A

Solution: $P = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $P\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \Rightarrow \begin{matrix} a - b = -1 \\ c - d = 2 \end{matrix}$

$P^2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow P\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} -a + 2b = 1 \\ -c + 2d = 0 \end{matrix}$

$\Rightarrow$ a = –1, b = 0, c = 4, d = 2

$|P - pI| = 0 \Rightarrow \begin{vmatrix} -1-p & 0 \\ 4 & 2-p \end{vmatrix} = 0$

$\Rightarrow p = -1, 2$

$\Rightarrow \; 5p_1 + 2p_2 = 5(2) + 2(-1) = 8$

Q38. Answer: N/A

Solution: The elements 0, –1, 1 can be arranged in the principal diagnol in 3! ways and remaining places can be filled 3! ways corresponding to each case $\left(\because a_{ij} = a_{ji}\right)$.

No of matrices = 3!3! = 36

Q39. Answer: N/A

Solution: $A = A^{-1} + 2I$

$|A - 2I| = |A^{-1}| = \frac{1}{|A|}$,

$A^2 - I = 2A \rightarrow |A^2 - I| = |2A|$

$|A - I|\,|A + I| = 2^3|A|$

$|A - I|\,|A + I|\,|A - 2I| = 2^3|A| \times \frac{1}{|A|} = 8$

Q40. Answer: N/A

Solution: $A^2 = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$

$\alpha^2 = 1$ and $\alpha + 1 = 5$

$\alpha \in f$

Q41. Answer: C

Solution: $|A| = e^{-t}\begin{vmatrix} 1 & \cos t & \sin t \\ 1 & -\cos t - \sin t & -\sin t + \cos t \\ 1 & 2\sin t & -2\cos t \end{vmatrix}$

$= e^{-t}[5\cos^2 t + 5\sin^2 t] \;\forall\, t \in R$

$= 5e^{-t} \neq 0 \;\forall\, t \in R$